1894MNRAS..54..388M

Note on the Computation of the Brightness of the Planets, with some Ephemerides for Observations of the Brightness of Mercury. By A. Marth.

In the *Nautical Almanacs* for the years 1835 to 1857 the times of the greatest brilliancy of *Venus* are predicted to the minute. Wondering at this precision in the prediction of such an occurrence, I tried, by computing a number of instances, to find out what hypothesis could have been adopted in the computations. The predicted times were found to be within a couple of days of the times computed upon the ordinary assumption that the diminution of brightness due to the defect of illumination depends simply on the proportion of the unilluminated portion to the whole of the disc; but what modification of this assumption had been introduced, or how the apparent precision had been obtained, I failed to detect. When, in 1853, I got the opportunity of making inquiries at the *Nautical Almanac* office, I learnt that nothing was known there about any hypothesis, but that the predicted times were very easily computed to a minute, as they were merely the times when the distance of *Venus* from the Earth reached a certain value. As this value had obviously been deduced upon the ordinary assumption, but by neglecting the eccentricities of the orbits, I pointed out the illusory character of the predicted times, and recommended to give in future merely the day, but the right day. Since the distances of *Venus* from the Sun and from the Earth r and Δ and the distance of the Sun from the Earth were given in the *N. A.*, the light-ratio of *Venus* could be found from them, without actually computing the phase angle v or the angle at *Venus* between r and Δ. For, assuming that the Sun lights up just one-half of the surface of the planet, or disregarding the small zone of doubtful breadth beyond, the light-ratio would be

$$\frac{\cos^2 \frac{1}{2} v}{r^2\Delta^2}, \text{ or } \frac{(r+\Delta)^2-R^2}{r^3\Delta^3};$$

so that by computing this value, or, if preferred,

$$\frac{(r+\Delta+R)\,(r+\Delta-R)}{r^3\Delta^3},$$

for some days near the time, the day during which this value reached a maximum would be the right one. This recommendation was adopted in the *N. A.* for 1858, and the computations have been made since in this manner; but from 1875 onwards the hour of the greatest brilliancy has again been inserted.

The logarithm of the light-ratio (0·4 of which represents

1894MNRAS..54..388M

a difference in brightness of one star-magnitude), according to the foregoing formula, the phase angle v, the angle e at the Earth between R and Δ, and the brightness of the planet according to Professor G. Müller's empirical formula for some days about the times of this year's maxima are the following, the magnitudes for the days in January having been already given by Müller himself in his paper "Ueber den grössten Glanz der Venus," *Astron. Nachr.*, vol. 132, p. 300:—

Greenwich Noon.	log. $\frac{(r+\Delta)^2-R^2}{r^3\Delta^3}$	v	e	Müller's Mag.
1894.		°	°	m
Jan. 9	1·07861	115·75	41·24	−4·3776
10	·07942	116·81	40·78	−4·3798
11	·07973	117·89	40·29	−4·3809
12	·07949	118·99	39·78	−4·3809
13	·07864	120·12	39·24	−4·3802
14	1·07714	121·28	38·68	−4·3782
Mar. 21	1·04501	120·92	38·40	−4·297
22	·04565	119·79	38·93	−4·297
23	·04570	118·68	39·43	−4·296
24	1·04519	117·60	39·90	−4·294

The interesting results which Professor Müller has deduced from his own measurements of the brightness of *Mercury* (in vol. viii. of the *Publicationen des Astrophysikalischen Observatoriums zu Potsdam*, and in his paper "Ueber die Lichtstärke des Planeten Mercur," in the *Astron. Nachr.* vol. 133, p. 47 ff.) render it very desirable that these results should be tested and the measurements, if possible, extended over a greater range of the phase angle. The smallest angular distance of *Mercury* from the Sun, at which Müller has succeeded in getting some measures, is 14°·7 (on February 14, 1881, in zenith distance 85°·1, the comparison being made with *Venus*), so that it will be feasible to extend the range, even if the difficulties in getting good measures nearer to the Sun cannot be surmounted. This may be seen by looking over the following ephemerides, which give the phase angle v, the angle e at the Earth between *Mercury* and the Sun, and the brightness of *Mercury* expressed in star magnitudes and computed (I.) by Müller's (second) empirical formula, and (II.) upon the makeshift assumption that the alteration of magnitude due to the defect of illumination is just double of what it should be according to the usual assumption, or, what amounts to the same, that the light-ratio is

$$\frac{\cos^4 \frac{1}{2}v}{r^2\Delta^2} \quad \text{instead of} \quad \frac{\cos^2 \frac{1}{2}v}{r^2\Delta^2}.$$

There is no justification for this makeshift assumption except the empirical one that, within the range of the phase angle between

1894MNRAS..54..388M

50° or 60° and 120°, it gives no greater discrepancies from the measured magnitudes than Müller's own formula, while outside this range the resulting magnitudes diverge rapidly, so that their comparison indicates sufficiently that for the recognition of the true hypothesis measurements over a considerably extended range are required. For the sake of the comparison the magnitudes derived from

$$\text{I.} = 5 \log \left(\frac{r\Delta}{r_0}\right) - 0{\cdot}901 + 0{\cdot}02838\,(v - 50^\circ) + 0{\cdot}0001223\,(v - 50^\circ)^2,$$

wherein $\log \frac{1}{r_0} = 0{\cdot}41218$

are carried some 10° beyond the limits of the phase angle within which they are valid. The magnitudes given by II. are

$$\text{II.} = 5 \log \left(\frac{r\Delta}{r_0}\right) - 1{\cdot}25 + 10 \log \sec \tfrac{1}{2}v.$$

The angles v and e are conveniently computed in the following way:—

In the plane triangle formed by $r\Delta\text{R}$, in which the angle v is opposite to R, and e opposite to r, we have

$$\cos v = \frac{r^2 + \Delta^2 - \text{R}^2}{2r\Delta} \text{ and } \cos e = \frac{\text{R}^2 + \Delta^2 - r^2}{2\text{R}\Delta}.$$

Hence

$$\tan^2 \tfrac{1}{2}v = \frac{1 - \cos v}{1 + \cos v} = \frac{\text{R} + r - \Delta}{\text{R} + r + \Delta} \cdot \frac{\Delta + \text{R} - r}{\Delta - (\text{R} - r)};$$

$$\tan^2 \tfrac{1}{2}e = \frac{1 - \cos e}{1 + \cos e} = \frac{\text{R} + r - \Delta}{\text{R} + r + \Delta} \cdot \frac{\Delta - (\text{R} - r)}{\Delta + \text{R} - r}.$$

Therefore, by computing

$$\tan^2 \tfrac{1}{2}\gamma = \frac{\Delta}{\text{R} + r} \text{ and } \tan^2 \tfrac{1}{2}\nu = \frac{\text{R} - r}{\Delta},$$

we get

$$\tan^2 \tfrac{1}{2}v = \frac{\cos \gamma}{\cos \nu} \text{ and } \tan^2 \tfrac{1}{2}e = \cos \gamma \, . \cos \nu,$$

and have also for check or control

$$r \sin v = \text{R} \sin e.$$

If $r > \text{R}$, as in the case of the asteroids, the corresponding formulæ are derived from the foregoing by exchanging v and r with e and R, so that, by computing

$$\tan^2 \tfrac{1}{2}\gamma = \frac{\Delta}{r + \text{R}} \text{ and } \tan^2 \tfrac{1}{2}\nu = \frac{r - \text{R}}{\Delta},$$

we get

$$\tan^2 \tfrac{1}{2}e = \frac{\cos \gamma}{\cos \nu} \text{ and } \tan^2 \tfrac{1}{2}v = \cos \gamma \, . \cos \nu.$$

1894MNRAS..54..391M

Greenwich Noon.		*Mercury.*		Magnitude.	
1894.		*v*	*e*	I.	II.
May	29	36°·11	10°·70	...	−0·95
	30	39·87	11·80	−1·10	−0·90
	31	43·52	12·86	−1·00	−0·85
June	1	47·05	13·88	−0·90	−0·79
	2	50·46	14·86	−0·80	−0·73
	3	53·76	15·80	−0·70	−0·66
	4	56·94	16·70	−0·60	−0·60
	5	60·00	17·56	−0·51	−0·53
	6	62·95	18·37	−0·42	−0·46
	7	65·80	19·14	−0·33	−0·39
	8	68·55	19·87	−0·24	−0·33
	9	71·20	20·55	−0·16	−0·26
	10	73·77	21·19	−0·08	−0·19
	11	76·26	21·79	0·00	−0·13
	12	78·68	22·34	0·07	−0·06
	13	81·03	22·84	0·14	0·00
	14	83·33	23·30	0·21	0·07
	15	85·57	23·71	0·28	0·13
	16	87·77	24·08	0·34	0·20
	17	89·93	24·40	0·40	0·26
	18	92·06	24·67	0·46	0·33
	19	94·17	24·89	0·52	0·39
	20	96·26	25·07	0·58	0·46
	21	98·33	25·19	0·64	0·52
	22	100·40	25·26	0·69	0·59
	23	102·47	25·28	0·75	0·66
	24	104·55	25·24	0·80	0·73
	25	106·65	25·15	0·86	0·81
	26	108·76	25·00	0·91	0·89
	27	110·89	24·79	0·97	0·97
	28	113·05	24·51	1·02	1·06
	29	115·24	24·18	1·08	1·16
	30	117·47	23·78	1·14	1·26
July	1	119·75	23·32	1·20	1·37
	2	122·08	22·79	1·27	1·49
	3	124·46	22·19	1·34	1·62
	4	126·90	21·52	1·41	1·77
	5	129·39	20·77	1·48	1·93
	6	131·96	19·96	1·56	2·10

1894MNRAS..54..391M

Greenwich Noon. 1894.		*v*	*e*	Magnitude. I.	Magnitude. II.
		°	°		
Aug.	1	132·68	15·93	1·38	1·95
	2	128·74	16·66	1·23	1·65
	3	124·76	17·30	1·08	1·38
	4	120·74	17·84	0·93	1·12
	5	116·67	18·27	0·79	0·89
	6	112·54	18·60	0·64	0·67
	7	108·35	18·83	0·50	0·47
	8	104·10	18·95	0·36	0·29
	9	99·80	18·97	0·22	0·12
	10	95·44	18·89	0·08	−0·04
	11	91·02	18·71	−0·05	−0·19
	12	86·55	18·44	−0·18	−0·32
	13	82·03	18·08	−0·30	−0·44
	14	77·47	17·64	−0·43	−0·55
	15	72·87	17·12	−0·55	−0·65
	16	68·26	16·52	−0·66	−0·74
	17	63·65	15·85	−0·76	−0·81
	18	59·06	15·12	−0·86	−0·87
	19	54·51	14·33	−0·95	−0·92
	20	50·02	13·49	−1·03	−0·96
	21	45·61	12·62	−1·11	−0·99
	22	41·31	11·72	−1·18	−1·00
	23	37·12	10·78	−1·24	−1·01
Nov.	17	130·69	13·73	0·97	1·46
	18	124·06	15·12	0·75	1·02
	19	117·71	16·32	0·55	0·68
	20	111·65	17·33	0·39	0·40
	21	105·89	18·16	0·24	0·18
	22	100·44	18·81	0·11	0·01
	23	95·28	19·32	0·01	−0·12
	24	90·41	19·70	−0·08	−0·22
	25	85·82	19·95	−0·16	−0·30
	26	81·49	20·10	−0·22	−0·36
	27	77·42	20·16	−0·27	−0·40
	28	73·59	20·13	−0·31	−0·42
	29	69·98	20·04	−0·35	−0·44
	30	66·58	19·88	−0·38	−0·45
Dec.	1	63·38	19·67	−0·40	−0·44
	2	60·36	19·42	−0·41	−0·44

1894MNRAS..54..391M

Greenwich Noon.	*v*	*e*	Magnitude. I.	II.
1894.	°	°		
Dec. 3	57·51	19·12	−0·43	−0·43
4	54·81	18·80	−0·44	−0·41
5	52·26	18·44	−0·45	−0·39
6	49·85	18·06	−0·45	−0·37
7	47·56	17·66	−0·45	−0·35
8	45·39	17·25	−0·46	−0·33
9	43·32	16·82	−0·46	−0·30
10	41·35	16·37	−0·46	−0·28
11	39·46	15·92	−0·46	−0·26
12	37·66	15·45	...	−0·24
13	35·93	14·97	...	−0·21
14	34·27	14·49	...	−0·19
15	32·68	14·01	...	−0·17
1895.				
Jan. 26	31·64	11·27	...	−0·67
27	34·41	11·96	...	−0·70
28	37·33	12·64	...	−0·73
29	40·42	13·31	−0·94	−0·75
30	43·68	13·97	−0·92	−0·78
31	47·13	14·60	−0·90	−0·79
Feb. 1	50·76	15·21	−0·87	−0·80
2	54·58	15·79	−0·84	−0·81
3	58·59	16·33	−0·79	−0·80
4	62·80	16·82	−0·74	−0·78
5	67·21	17·25	−0·68	−0·75
6	71·81	17·61	−0·61	−0·71
7	76·60	17·90	−0·52	−0·64
8	81·57	17·09	−0·42	−0·56
9	86·71	18·17	−0·31	−0·46
10	92·01	18·14	−0·19	−0·33
11	97·44	17·99	−0·05	−0·17
12	103·00	17·70	0·10	0·02
13	108·67	17·26	0·26	0·24
14	114·43	16·66	0·44	0·50
15	120·25	15·91	0·63	0·81
16	126·12	14·99	0·84	1·18
17	132·02	13·91	1·06	1·61
18	137·90	12·68	...	2·11
19	143·73	11·31	...	2·70

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